A set is a collection of numbers.

We indicate sets by braces or capital letters of the English alphabet.

A set is either a finite set or an infinite set. Some examples are as follows:

a. \{1, 2, 3\}
b. \{3\}
c. \{1, 2, 3, 4, 5, \ldots\}
d. \{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}
e. \{2, 4, 6, 8, 10\}

Note: The three dots, \ldots, ellipsis, indicate that the list of numbers continues forever.

The sets a., b., and e. are finite sets. The set b. is called the empty set; it does not have an element. We also indicate the empty set by the Greek letter phi, \emptyset. (So, do not write \emptyset for zero.)

Often a number must satisfy one or more properties to be an element of a set. In this case, set-builder notation is used. Three examples are:

a. \{x | x \text{ is a prime no. less than } 8\}
   (This is read, "The set of no. s \(x\) such that \(x\) is a prime no. less than 8.)

b. \{x | x \text{ is a natural no. less than 1}\}

c. \{x | x \text{ is an even positive integer}\}

def A set \(A\) is equal to a set \(B\), denoted \(A = B\), if they have the same elements.
A set \( A \) is a subset of a set \( B \), if every element of \( A \) is an element of \( B \).

**Comment:** The empty set is a subset of every set.

**Example:** Let \( A = \{1, 2, 3, 4\} \) and \( B = \{0, 1, 2, 3, 4, 5\} \). Since every element of \( A \) is an element of \( B \), \( A \) is a subset of \( B \).

**Example:** Let \( A = \{2, 4, \pi\} \) and \( B = \{1, 2, 3, 4, \ldots\} \). Since \( \pi \) is not an element of \( B \), \( A \) is not a subset of \( B \).

**Example:** Let \( A = \{2, 4\} \) and \( B = \{2, 4, 5\} \). Since the empty set is a subset of every set, \( A \) is a subset of \( B \).

**Example.**

**Given:** \( A = \{1, 2, 3\} \)

**List the subsets of**: \( A \).

**Solution.**

The subsets of \( A \) are: \( \emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\} \).

**Example.**

**Given:** \( A = \{1, 2, 3\} \)

**List the subsets of**: \( A \).

**Solution.**

The subset of \( A \) are: \( \emptyset, A \)

\[ {\{1, 2\}, \{1, 3\}, \{2, 3\} \} \]
We now consider some important subsets of the set of real numbers. Then we construct a Real No. Tree Diagram.

**Def. Natural No.s**

\[ 1, 2, 3, 4, 5, \ldots \]

**Def. A prime no. is a natural no. that is greater than one and can be divided evenly only by 1 and itself.**

\[ 2, 3, 5, 7, 11, 13, 17, \ldots \]

**Def. A composite no. is a natural no. that is greater than one that is not a prime no.**

\[ 4, 6, 8, 9, 10, 12, 14, 15, \ldots \]

**Def. If we include 0 with the set of natural no.s, we have the set of whole numbers.**

\[ 0, 1, 2, 3, 4, 5, \ldots \]

**Def. The set of integers consist of the whole no.s and the opposite of every natural no.**

\[ \ldots -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, \ldots \]

**Def. Those integers divisible by 2 are called even integers.**

\[ \ldots -6, -4, -2, 0, 2, 4, 6, \ldots \]

**Def. Those integers not divisible by 2 are called odd integers.**

\[ \ldots -5, -3, -1, 1, 3, 5, \ldots \]
The set of rational numbers is defined as follows:
\[ \frac{p}{q} \text{ where } p \text{ and } q \text{ are integers and } q \neq 0. \]

(These set of fractions)

Examples:
\[ \frac{1}{3}, -\frac{5}{4}, 8, \frac{16}{3}, -5 \]

Now, every rational no. in fractional form can be changed to decimal form.

Example:
\[ \frac{3}{4} \text{ can be changed to a terminating decimal.} \]

\[
\begin{array}{c}
0.75 \\
4 | 3.00 \\
\underline{2.8} \\
0.20 \\
\underline{0} \\
\end{array}
\]

\[ \frac{3}{4} = 0.75 \]

We divide the denominator 4 into the numerator 3.

Since the remainder is zero, 0.75 is a terminating decimal.

Example.
\[ \frac{1}{3} \text{ can be changed to a repeating decimal.} \]

\[
\begin{array}{c}
0.333 \ldots \\
3 | 1.0000 \ldots \\
\underline{9} \\
10 \\
\frac{9}{9} \\
1 \ldots \\
\end{array}
\]

\[ \frac{1}{3} = 0.333\ldots = 0.\overline{3} \]

The remainder is not zero.

(cont.)
Comment:
Every rational no. in fractional form can be written as either a terminating decimal or a repeating decimal. In a similar manner, a terminating decimal or repeating decimal can be written as a rational no. in fractional form.

An alternative def. of the set of rational nos. is as follows:
\[ \exists x \in \mathbb{R} \] \[ x \] is a repeating or a terminating decimal.

**def.** Irrational Numbers.

Examples. \( \sqrt{2}, \pi, \sqrt{5} \)

\[ \exists x \in \mathbb{R} \] \[ x \] is neither a repeating decimal nor a terminating decimal.

**def.** The set of real nos. consists of all rational nos. and all irrational nos.

**Real No. Tree Diagram**

```
Real Nos
   /  \  
\  /    \ 
Irrational Nos  Rational Nos
   \     /
       \  
       \ /
       \=/
         \ 
Integers
     /  
Whole Nos
     /  
Natural Nos
     /  
Prime Nos Composite Nos
```

(Cont.)
Example.
Given: \(\frac{1}{2}, 0.38, -3, 0, 1, -1\frac{3}{8}, 8, \sqrt{15}\)
Which nos. in the given set are:

a. prime nos.
b. composite nos.
c. nat'l nos.
d. whole nos.
e. integers
f. rat'l nos.
g. irrat'l nos.
h. real nos.

Examples.
List the elements in each set.

a. \(3 \times 1\) \(x\) is a composite no. between 3 and 15?
b. \(3 \times 1\) \(x\) is a natural no. that is neither prime nor composite?
c. \(3 \times 1\) \(x\) is an even prime no.? 

Examples.
Indicate true or false.

a. The smallest prime no. is one.
b. All integers are rational nos.
c. The set of prime nos. is a subset of the set of odd integers.
d. Every rational no. is a real no.
e. No irrational no. is an integer.
Sets of numbers can be represented geometrically as points on a number line. The point on the no. line associated with each no. is called the graph of that no. The no. is called the coordinate of its corresponding point. For every real no. there corresponds a point on the no. line, and for every point on the no. line there corresponds a point.

No.s that are greater than zero are positive no.s. No.s that are less than zero are negative no.s. The no. zero is neither positive nor negative.

Example:
Given: $-3, -\frac{1}{2}, 0, \frac{5}{4}, 2$.
Graph the given points.
Math 123A
[1,7] (cont)

Symbols

a. \( \approx \) "is approximately equal to"

b. \( = \) "is equal to"

c. \( \neq \) "is not equal to"

d. \( > \) "is greater than"

e. \( \geq \) "is greater than or equal to"

f. \( < \) "is less than"

g. \( \leq \) "is less than or equal to"

Examples.

a. The irrational no. \( \pi = 3.141592654... \)

\( \pi \approx 3.14 \) or \( \pi \approx \frac{22}{7} \)

b. A variable, such as \( x \), represents a real no.

\( x = 3 \)

c. Clearly, \( 8 \neq 7 \).

d. The inequality symbols are used below.

1. \( -3 < 1 \)

2. \( x \geq 2 \)

3. \( 8 > 0 \)

4. \( y \leq -1 \)

(cont)
The intersection and union of two sets.

**def** If the elements of some set $A$ are united with the elements of some set $B$, the union of set $A$ and set $B$ is formed. The union of set $A$ and set $B$ is denoted as

\[ A \cup B \quad \text{"the union of set } A \text{ and set } B \"

The elements in $A \cup B$ are either elements of set $A$, or elements of set $B$, or elements of both set $A$ and set $B$.

\[ A \cup B = \{ x \mid x \text{ is an element of } A \text{ or } x \text{ is an element of } B \} \]

**def** The set of elements that are common to set $A$ and set $B$ is called the intersection of $A$ and $B$. The intersection of set $A$ and set $B$ is denoted as

\[ A \cap B \quad \text{"the intersection of set } A \text{ and set } B \"

The elements of $A \cap B$ are those elements that are in both set $A$ and set $B$.

\[ A \cap B = \{ x \mid x \text{ is an element of } A \text{ and } x \text{ is an element of } B \} \]

**def** If $A$ and $B$ have no elements in common, $A \cap B = \emptyset$. When the intersection of two sets is the empty set, we say that the two sets are disjoint.

(cont.)
$\text{Math 123A}$

[1, 7] (cont.)

$\text{A \cup B and A \cap B}$ (cont.)

Examples.

1. Given: $A = \{1, 2, 3, 4\}$ and $B = \{4\}$
   a. $A \cup B = \{1, 2, 3, 4\}$
   b. $A \cap B = \emptyset$

2. Given: $A = \{\pi, 5, \sqrt{2}\}$ and $B = \{\pi, 5, \sqrt{2}\}$
   a. $A \cup B = \{\pi, 5, \sqrt{2}\}$
   b. $A \cap B = \{\pi, 5, \sqrt{2}\}$

3. Given: $A = \emptyset$ and $B = \{\frac{1}{2}, -1, 0\}$
   a. $A \cup B = B = \{\frac{1}{2}, -1, 0\}$
   b. $A \cap B = \emptyset$

4. Given: $A = \{0, 1, 2, 3, 4, \ldots\}$ and $B = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$
   a. $A \cup B = B = \{\ldots, -2, -1, 0, 1, 2, \ldots\}$
   b. $A \cap B = A = \{0, 1, 2, 3, \ldots\}$

5. Given: $A =$ rational nos., $B =$ irrational nos., $R =$ real nos.
   a. $A \cup B = R$
   b. $A \cap B = \emptyset$
   c. $A \cup R = R$
   d. $B \cup R = R$
   e. $A \cap R = A$

(cont.)
An interval is a set of real nos. as listed below.

1. \((a, b) = \{x \mid a < x < b\}\)
2. \([a, b) = \{x \mid a \leq x < b\}\)
3. \((a, b] = \{x \mid a < x \leq b\}\)
4. \([a, b] = \{x \mid a \leq x \leq b\}\)
5. \((a, +\infty) = \{x \mid a < x < +\infty\}\)
6. \([a, +\infty) = \{x \mid a \leq x < +\infty\}\)
7. \((-\infty, b) = \{x \mid -\infty < x < b\}\)
8. \((-\infty, b] = \{x \mid -\infty < x \leq b\}\)
9. \((-\infty, +\infty) = \{x \mid -\infty < x < +\infty\}\)

Note: \(\mathbb{R}\) is the set of real nos.

Examples:

Graph:

- a. \((1, 3)\), \([-3, +\infty)\), \((-\infty, 0)\), \([-2, 1]\), \([4, 4]\)
- b. \([-4, -2, 0, 2, 4]\)
- c. \([-6, -4, -2, 0, 2]\)
- d. \([-2, -1, 0, 1, 2]\)
- e. \([0, 2, 4, 6]\)

(Cont.)
Graph subsets of the Real Nos

Examples

a. Graph \( X < -3 \) or \( X > 1 \).

Construct a number line.
Graph all real nos. less than \(-3\). \{ use only one no. line. \}
Graph all real nos. greater than \(1\).

\[ \text{NOTE: } X < -3 \text{ is the interval } (-\infty, -3), \text{ and } X > 1 \text{ is the interval } (1, \infty). \]

2. The graph above is \((-\infty, -3) \cup (1, \infty)\)

b. Graph \( X < -3 \) and \( X > 1 \).

Construct a no. line.
Graph \( X < -3 \) as in a.
Graph \( X > 1 \) as in a.

Observe that \( X < -3 \) and \( X > 1 \) do not have common real nos. So, their intersection is the empty set.
Hence, there is no graph for \((-\infty, -3) \cap (1, \infty) = \emptyset\).

C. Graph \( X \geq -2 \) and \( X < 5 \).
Absolute Value of a Real No.

Def: The absolute value of a real no. \( x \), denoted as \( |x| \), is the distance on a no. line from 0 to the point with the coordinate \( x \).

\[
\begin{array}{cccccc}
-2 & -1 & 0 & 1 & 2 & 3 \\
\hline
\end{array}
\]

\[|x|\] absolute value of \( x \)

An equivalent def. is as follows:

For any real no. \( x \),

\[
|x| = \begin{cases} 
  x & \text{if } x \geq 0 \\
  -x & \text{if } x < 0
\end{cases}
\]

Comments:

1. If \( x \) is a positive no. or 0, then \( x \) is its own absolute value.
2. If \( x \) is a negative no., then \( -x \) (which is positive) is its absolute value.

Examples:

a. \(|5| = 5\)
b. \(|0| = 0\)
c. \(|-6| = -(-6) = 6\)
d. \(-|7-8| = -|-1| = -1\)
e. \(-|-3| = -3\)

(Cont.)
Math 123A
L1,3 (cont.)

Absolute Value (cont.)

3 properties of the absolute value are:

1. For any real no. $x$, $|x| \geq 0$

2. If $x = 0$, $|x| = 0$.

3. For any real no. $x$, $|x| = |-x|$.
   
   (e.g., $|−2| = |2| = 2$.)
[1.7] Properties of Real Nos

Real nos. can be added, subtracted, multiplied, and divided. And, we might think that these are 4 separate operations. However, subtraction can be written as an addition, and division can be written as multiplication. Some examples are:

a. \( 5 - 3 = 5 + (-3) \)
b. \( a - b = a + (-b) \) where \( a \) and \( b \) are any real nos.
c. \( \frac{5}{7} = 5 \cdot \left( \frac{1}{7} \right) \)
d. \( \frac{p}{q} = p \cdot \left( \frac{1}{q} \right) \) where \( p \) and \( q \) are any real nos, except \( q \neq 0 \).

So, there are only two operations (considering the above 4). One operation is addition, which we denote by \( + \), and the other operation is multiplication, which we denote by \( \cdot \), \( \times \), or \( ( ) \). These two operations are binary operations. That is, for them to make sense requires two real nos. Suppose we have

a. \( 5 + \_ = \)

b. \( 7 \cdot \_ = \)

Without a real no. in the blanks the two operations cannot be performed. On the other hand,

a. \( 5 + 11 = 16 \)
b. \( 7 \cdot 6 = 42 \).

(cont)
We consider properties of addition, multiplication, and distributive, respectively.

**Addition Properties**

1. If a and b are real nos., then $a+b$ is a real no.
   
   **Examples:**
   
   a. $3+7=10$
   b. $8+(-1)=7$
   c. $6+0=6$

   This is the **closure property of addition**.

   **Note:** If the sum of 2 real nos. is not a real no., we would have difficulty working or solving problems.

2. If a and b are real nos., then $a+b=b+a$.
   
   The order that we add two real nos. does not matter.
   
   **Examples:**
   
   a. $5+6=6+5=11$
   b. $1+3=3+1=4$
   c. $8+(-3)=(-3)+8=5$

   This is the **commutative property of addition**.

3. If $a$, $b$, and $c$ are real nos., then $(a+b)+c=a+(b+c)$.
   
   We can group two real nos. together. Find their sum. Then add this to another real no. Observe that the order of $a$, $b$, and $c$ did not change.
   
   **Examples:**
   
   a. $(5+6)+10=5+(6+10)=21$
   c. $(8+2)+6=8+(2+6)=16$
   b. $(3+(-1))+4=3+((-1)+4)=6$

   This is the **associative property of addition**.
4. If \( a \) is a real no., then \( a + 0 = a \). The no. zero is the identity element of addition.

Examples:
\[
\begin{align*}
a. & \quad \pi + 0 = \pi \\
b. & \quad \frac{1}{2} + 0 = \frac{1}{2} \\
c. & \quad -3 \frac{1}{2} + 0 = -3 \frac{1}{2}
\end{align*}
\]
This is the identity property of addition.

5. For every real no. \( a \) there is a real no. \( b \) such that \( a + b = 0 \). \( b \) is the opposite of \( a \), i.e. \( b = -a \). \( b \) is called the additive inverse of \( a \).

Examples:
\[
\begin{align*}
a. & \quad 5 + (-5) = 0 \quad b = -5 \\
b. & \quad -\frac{2}{3} + \frac{2}{3} = 0 \quad b = \frac{2}{3} \\
c. & \quad 1 + (-1) = 0 \quad b = -1
\end{align*}
\]
This is the inverse property of addition

**Multiplication Properties**

1. If \( a \) and \( b \) are real nos., then \( a \cdot b \) is a real no.

Examples:
\[
\begin{align*}
a. & \quad (10)(7) = 70 \\
b. & \quad (-3)(-5) = 15 \\
c. & \quad (-\frac{1}{2})(\frac{4}{9}) = -\frac{2}{9}
\end{align*}
\]
This is the closure property of multiplication.
2. If $a$ and $b$ are real nos, $a \cdot b = b \cdot a$.
   Examples.
   a. $7 \cdot 3 = 3 \cdot 7 = 21$
   b. $(-3) \cdot 6 = (-6) \cdot (-3) = 18$
   c. $(16) \cdot (-2) = (-2) \cdot (16) = -32$

   This is the **commutative property of multiplication**

3. If $a$, $b$, and $c$ are real nos, $(a \cdot b) \cdot c = a \cdot (b \cdot c)$.
   This is the **associative property of multiplication**.
   **Note:** You compare with the associative property of addition.

   Examples.
   a. $(7 \cdot 2) \cdot 3 = 7 \cdot (2 \cdot 3) = 42$
   b. $(-1) \cdot (8 \cdot 5) = ((-1) \cdot 8) \cdot 5 = -40$
   c. $(-2 \cdot 3) \cdot (-4) = -2 \cdot (3 \cdot (-4)) = 24$

4. Given: a real no. "$a"
   Then $a \cdot 1 = a$. The no. 1 is the **identity element of multiplication**.

   Examples.
   a. $16 \cdot 1 = 16$
   b. $(-7) \cdot 1 = -7$
   c. $(-1) \cdot 1 = -1$

   This is the **identity property of multiplication**
5. The no. one is the identity element of multiplication.
   (See 4. for examples.)

   Given: a real no. a
   Does there exist a real no. b such that \( a \cdot b = 1 \)
   answer.

   For every real no. \( a \), except zero, there is a real no.
   \( b = \frac{1}{a} \) such that \( a \cdot b = a \left( \frac{1}{a} \right) = 1 \).

   Examples.
   a. \( 5 \left( \frac{1}{5} \right) = 1 \)
   b. \( \frac{2}{3} \left( \frac{3}{2} \right) = \frac{2}{3} \cdot \frac{3}{2} = 1 \)
   c. \( -4 \left( \frac{1}{4} \right) = 1 \)

   However, \( 0 \left( \frac{1}{0} \right) = ? \) Hence, we exclude zero.

   The no. \( b = \frac{1}{a} \), for \( a \neq 0 \), is called the multiplicative
   inverse of \( a \).

   This is the inverse property of multiplication.

Distributive Property

   Let \( a \), \( b \), and \( c \) be real no.s. Then \( a(b+c) = ab + ac \).
   Multiplication is distributive over addition.

   Examples.
   a. \( -2(-7+3) = (-2)(-7) + (-2)(3) \)
   b. \( 8(x+2) = 8x + 16 \quad x \) is a variable
Math 123A

Exam 1

- \([a_i, 0]\)
- \([b_1, 7]\)
- Order of Operations (pp. 29-30)
  1. exponential expressions
  2. perfect squares
  3. square roots
  4. real no.s

- Exercises:
  a. \([a_i, 7]\) pp. 11-12: 1-40, 73-104
  b. \([a_i, 7]\) pp. 69-70: 7-20, 41-90
  c. Order of Operations: Pg. 36: 51-86

- Material Covered in Class